Local Curvature Obstructions and Beilinson–Bloch Height Pairings

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Abstract

An obstruction approach to Beilinson–Bloch heights separates the construction of a local Green pair from the removal of its curvature. We formulate the obstruction in the actual Bott–Chern space, prove the curvature-correction argument on the prescribed support complement, and explain the vanishing mechanisms supplied by algebraic equivalence and Chow correspondences. Over a number field, the local constructions first give a pairing on the intersection of the obstruction kernels. We prove a local-to-global criterion and identify the exact additional assertion needed to obtain the full complementary homologically trivial Chow groups: vanishing on localizations of global classes in both degrees. Finite period descriptions require a separate comparison with this original obstruction. The remaining construction, vanishing and comparison problems are stated explicitly.

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1 The height pairing and the vanishing problem

Let F be a number field, and let X/F be smooth, projective and geometrically integral of dimension $n \geq 1$. Put

$$V^k(X) = \mathrm{CH}^k(X)_{\mathbf{Q},\mathrm{hom}}, \qquad r + s = n + 1, \qquad 1 \leq r, s \leq n.$$

Here homological triviality is geometric. Over a number field it can be defined by the rational Betti cycle class after an embedding into $\mathbf{C}$; over a finite extension of $\mathbf{Q}_p$ we use the geometric rational ℓ-adic cycle class, with $\ell \neq p$.

The objective is a canonical height pairing

$$h_X : V^r(X) \times V^s(X) \longrightarrow \mathbf{R} \tag{1.1}$$

constructed from local Green forms and independent of rational representatives. The complementary degree condition makes disjoint representatives possible. The local Green integral is then a secondary intersection number, and the product formula is responsible for its descent to Chow groups.

The present approach begins with a rational Chow homomorphism at each finite place,

$$\mathfrak{o}^k_{X_{F_v},F_v} : \mathrm{CH}^k(X_{F_v})_{\mathbf{Q}} \longrightarrow H^{k,k}_{\mathrm{BC,act}}(X_{F_v}),$$

whose target and construction are specified in Section 2. Define

$$\mathcal{H}^k(X) = \{z \in V^k(X) : \mathfrak{o}^k_{X_{F_v},F_v}(z_{F_v}) = 0 \text{ for every finite place } v\}. \tag{1.2}$$

The conditional construction gives a pairing on $\mathcal{H}^r(X) \times \mathcal{H}^s(X)$. Its extension to the full domain in (1.1) is therefore governed by the following problem.

Problem 1.1 (Vanishing on global cycle classes)**.** For every such X/F, prove

$$\mathfrak{o}^k_{X_{F_v},F_v}(z_{F_v}) = 0 \quad \text{for all } z \in V^k(X), \quad v \nmid \infty, \quad k = r, s. \tag{1.3}$$

The quantifiers in this problem matter. A solution is needed in both complementary degrees. It concerns only classes obtained by localizing a global class; it does not require vanishing on every homologically trivial class over F_v. General local vanishing is a sufficient, stronger answer to this particular vanishing problem.

The complete local construction and the global residue calculation are developed in the companion manuscripts [2, 3]. Here we isolate their mathematical premises and give the reductions needed for this route. The geometric deductions in Section 3 depend on the actual-form, model and correspondence hypotheses stated there. The proof of the passage from local heights to (1.1) is in Section 4.1.

2 Curvature correction on the prescribed complement

Fix a finite extension $K/\mathbf{Q}_p$ and an absolute value extending $|p|_p = p^{-1}$. Every further finite extension carries this absolute value extended from K. Let X/K be smooth, projective and geometrically integral of dimension n. We use Mihatsch's sheaf $\mathcal{B}^{a,b}$ of ordinary actual δ-forms on X^{an} [1], and write $D = d'd''$. The adjective actual specifies a section of this sheaf, rather than only a current or a formal expression in a chosen tropical presentation.

For a rational cycle $Z = \sum_i a_i[Z_i]$ with distinct prime components and $a_i \neq 0$, put

$$|Z| = \bigcup_i Z_i, \qquad U_Z = X^{\mathrm{an}} \setminus |Z|^{\mathrm{an}}.$$

The support is taken after collecting equal components. Integration over Z retains its coefficients and the native field multiplicities.

Definition 2.1 (The actual Bott–Chern space)**.** For $1 \leq k \leq n$, let

$$H^{k,k}_{\mathrm{BC,act}}(X) = \frac{\{\omega \in \mathcal{B}^{k,k}(X^{\mathrm{an}}) : d'\omega = d''\omega = 0\}}{D\mathcal{B}^{k-1,k-1}(X^{\mathrm{an}})}.$$

This is an algebraic quotient of real vector spaces. A zero class means the existence of a global actual primitive.

Hypothesis 2.2 (The curvature obstruction and Green pairs)**.** For every variety and degree used in the argument, there is a rational-linear Chow homomorphism

$$\mathfrak{o}^k_{X,K} : \mathrm{CH}^k(X)_{\mathbf{Q}} \longrightarrow H^{k,k}_{\mathrm{BC,act}}(X)$$

with the following property. Each prescribed rational codimension-k cycle Z has forms

$$G_Z \in \mathcal{B}^{k-1,k-1}(U_Z), \qquad \Omega_Z \in \mathcal{B}^{k,k}(X^{\mathrm{an}}),$$

where Ω_Z is separately closed and $[\Omega_Z] = \mathfrak{o}^k_{X,K}([Z])$, such that

$$\int_{U_Z} G_Z \wedge D\eta + \int_{Z^{\mathrm{an}}} \eta = \int_{X^{\mathrm{an}}} \Omega_Z \wedge \eta \quad \big(\eta \in \mathcal{B}_c^{n-k,n-k}(X^{\mathrm{an}})\big). \tag{2.1}$$

All these integrals are defined. Actual Stokes applies to global actual forms, and removing a proper algebraic closed subset does not change the relevant top-degree integrals. The obstruction is computed by model Chern-character forms and is compatible with the pullbacks used below, including rational fibres of families.

In the supported-complex construction, Ω_Z is the degree-k model Chern character of a complex resolving Z. The localized character determines the residue in (2.1). Independence of the model and of the chosen resolution is what makes its curvature class a homomorphism on the Chow group. These are substantive parts of Hypothesis 2.2; their construction is the subject of [2, Sections 3–5].

Proposition 2.3 (Removal of curvature)**.** *Assume Hypothesis 2.2. If $\mathfrak{o}^k_{X,K}([Z]) = 0$, then there exists*

$$\gamma_Z \in \mathcal{B}^{k-1,k-1}(U_Z)$$

satisfying

$$\int_{U_Z} \gamma_Z \wedge D\eta = -\int_{Z^{\mathrm{an}}} \eta \quad \big(\eta \in \mathcal{B}_c^{n-k,n-k}(X^{\mathrm{an}})\big). \tag{2.2}$$

Proof. The vanishing of $[\Omega_Z]$ in Definition 2.1 gives a global form $\Psi_Z \in \mathcal{B}^{k-1,k-1}(X^{\mathrm{an}})$ with $D\Psi_Z = \Omega_Z$. Set

$$\gamma_Z = G_Z - \Psi_Z|_{U_Z}.$$

For every stated test, Stokes gives

$$\int_{U_Z} \Psi_Z \wedge D\eta = \int_{X^{\mathrm{an}}} \Psi_Z \wedge D\eta = \int_{X^{\mathrm{an}}} \Omega_Z \wedge \eta.$$

Subtract this equality from (2.1). The primitive is defined on all of X^{an}, so the resulting form has exactly the required domain U_Z. □

This is a sufficient obstruction criterion. A strong Green form constructed by another method need not come with a global actual primitive for the particular Ω_Z selected here. Identifying those two constructions is a separate comparison problem.

2.1 Descent of a curvature primitive

Proposition 2.4 (Finite trace)**.** *Let $q : Y \to X$ be finite dominant of native degree d. Suppose that an actual trace is defined on the forms below, commutes with D, and satisfies*

$$\mathrm{Tr}_q(q^*\Omega) = d\Omega.$$

If β belongs to its domain and $D\beta = q^*\Omega$, then

$$\Psi = d^{-1}\,\mathrm{Tr}_q\,\beta \quad \textit{satisfies} \quad D\Psi = \Omega.$$

Proof. Commutation with D and the degree formula give

$$D\Psi = d^{-1}\operatorname{Tr}_q(D\beta) = d^{-1}\operatorname{Tr}_q(q^*\Omega) = \Omega.$$

□

For a constant extension L/K, the factor is the full degree $[L : K]$, with the absolute value extended from K. The trace must act on the actual form represented by β: relations among presentations must remain relations after trace. The restricted construction in [2, Section 4] uses sums of scalar–curvature expressions $u_0 Du_1 \cdots Du_m$. It must prove both presentation independence and adjunction against every actual test. A trace on a free list of expressions is not yet the operator in Proposition 2.4.

3 Geometric methods for proving vanishing

The obstruction is a linear map on a Chow group. Algebraic equivalence gives an additive parameter space for its values. A Chow decomposition expresses a class through source varieties on which vanishing is known. A numerical model correction provides a third way of producing the needed primitive. These mechanisms can be used independently or combined.

3.1 Algebraically trivial cycles

Lemma 3.1 (Compact groups and locally constant homomorphisms)**.** *Let A be a compact topological abelian group, and let W be a torsion-free abelian group. If a homomorphism $\varphi : A \to W$ has a kernel containing an identity neighbourhood, then $\varphi = 0$.*

Proof. Choose an open identity neighbourhood $U \subseteq \ker\varphi$. Then $\ker\varphi$ is open, since it is a union of translates of U. Its cosets form an open cover of A, so compactness implies that the quotient $A/\ker\varphi$ is finite. The image of φ is therefore finite. Every element of that image is torsion, whereas W is torsion-free. □

Hypothesis 3.2 (Constancy on nearby rational fibres)**.** For smooth separated S/K, a point $a_0 \in S(K)$ and an actual form ω on $(X \times S)^{\mathrm{an}}$, there is an ordinary neighbourhood U of a_0 such that

$$s_a^*\omega = s_{a_0}^*\omega \qquad (a \in U \cap S(K)).$$

The obstruction of a Chow family is represented by a global model Chern form whose fibre restriction represents the obstruction of that fibre.

Theorem 3.3 (Vanishing on algebraically trivial classes)**.** *Assume Hypotheses 2.2 and 3.2. Then*

$$\mathfrak{o}^k_{X,K}\big(\mathrm{CH}^k(X)_{\mathbf{Q},\mathrm{alg}}\big) = 0.$$

Here algebraic triviality is allowed over $\overline{K}$. Every prescribed representative of such a class consequently has the Green form of Proposition 2.3.

Proof. Let z be geometrically algebraically trivial. Choose a finite extension L/K over which the finite collection of curves, degree-zero divisors and correspondences expressing z_L by algebraic equivalence is defined. Normalized Poincaré bundles on these curves and their Jacobians give a product of Jacobians J/L and a Chow family Θ on $X_L \times J$ whose fibre map

$$J(L) \longrightarrow \mathrm{CH}^k(X_L)_{\mathbf{Q}}$$

is additive, sends zero to zero, and takes some $t \in J(L)$ to z_L. Rational coefficients are included in this family. This is the curve-parameter construction used in [2, Section 9.1].

Put $A = \mathrm{Res}_{L/K} J$. Pull Θ to $X_L \times A_L$ using evaluation $A_L \to J$, then push it algebraically along $X_L \times A_L \to X \times A$. The resulting family Θ^N is additive on fibres. At the point $a \in A(K) = J(L)$ corresponding to t, proper pushforward and refined fibre restriction give

$$(\Theta^N)_a = [L : K]z.$$

Thus

$$\varphi : A(K) \longrightarrow H^{k,k}_{\mathrm{BC,act}}(X), \qquad b \longmapsto \mathfrak{o}^k_{X,K}((\Theta^N)_b)$$

is a homomorphism. By Hypothesis 3.2, a model form representing the family obstruction has constant fibre restriction near zero. Since the zero fibre has zero class, $\ker \varphi$ contains an identity neighbourhood.

The group $A(K)$ is compact because A is an abelian variety over the local field K. The target is a real vector space. Lemma 3.1 gives $\varphi = 0$, hence $[L : K]\mathfrak{o}^k_{X,K}(z) = 0$. Rational linearity permits division by $[L : K]$. Applying Proposition 2.3 to the original cycle representative completes the proof. □

After this vanishing, the unresolved local map factors through

$$\mathrm{CH}^k(X)_{\mathbf{Q},\mathrm{hom}} / \mathrm{CH}^k(X)_{\mathbf{Q},\mathrm{alg}}. \tag{3.1}$$

The required assertion is that its image under $\mathfrak{o}^k_{X,K}$ is zero. It does not require the entire quotient (3.1) to vanish.

3.2 Chow correspondences and finite descent

A correspondence of degree a from a smooth projective V to a smooth projective W is a class in $\mathrm{CH}^{\dim V + a}(V \times W)_{\mathbf{Q}}$; it sends codimension k to codimension $k + a$.

Hypothesis 3.4 (Preservation of the obstruction kernels)**.** Rational Chow correspondences preserve the obstruction kernels in their specified degrees. This includes proper algebraic pushforward and finite constant extensions. For a variety defined over a finite L/K, the assertion may be used in the native K-theory, with the original absolute value and full degree. Homological triviality is imposed on every geometric component.

Proposition 3.5 (Transfer through a Chow decomposition)**.** *Assume Hypothesis 3.4. Let L/K be finite, and suppose that correspondences*

$$u_j : X_L \rightsquigarrow V_j, \qquad v_j : V_j \rightsquigarrow X_L$$

have opposite degrees and satisfy

$$\sum_j (v_j)_*(u_j)_* = \lambda \,\mathrm{id} \quad \text{on } \mathrm{CH}^k(X_L)_{\mathbf{Q},\mathrm{hom}}, \qquad \lambda \in \mathbf{Q}^\times.$$

If $(u_j)_* z_L$ has zero obstruction in the native K-theory for every j, then $\mathfrak{o}^k_{X,K}(z) = 0$.

Proof. Kernel preservation puts each $(v_j)_*(u_j)_* z_L$ in the kernel on X_L. Their sum is λz_L, so λz_L has zero obstruction. Push forward along $\pi : X_L \to X$. The Chow projection formula gives

$$\pi_*(\lambda z_L) = \lambda [L : K] z.$$

This class is again in the kernel. The scalar $\lambda[L : K]$ is nonzero, so rational linearity proves the assertion. □

For example, a diagonal identity through Tate twists of abelian varieties reduces the obstruction to the corresponding abelian groups. The identity must be an identity of rational Chow correspondences, with the indicated degree shifts. A cohomological decomposition alone does not give the displayed equality on homologically trivial Chow classes. The companion paper [4, Sections 3–8] constructs such identities and records the hypotheses for the resulting families.

3.3 Vertical correction on a model

The algebraic part of a numerical correction has a short form. Let $\mathcal{X}/R$ be a regular projective flat model over a complete discrete valuation ring, of relative dimension n, with strictly semistable special fibre Y. Write $i : Y \hookrightarrow \mathcal{X}$. Suppose component cycle-class groups $A^k(Y)$ and $A_j(Y)$ are supplied, with the restriction, Gysin and cap-product maps used below. Here $A^k(Y)$ is a subgroup of the direct sum of the geometric cycle-class groups of the smooth irreducible components of Y; its zero element therefore has zero class on every component.

Proposition 3.6 (Correction of a cycle extension)**.** *Suppose that*

$$A_{n+1-k}(Y) \xrightarrow{i^*i_*} A^k(Y) \xrightarrow{\cap[Y]} A_{n-k}(Y)$$

is exact at $A^k(Y)$, that $A_{n-k}(Y)$ injects into the corresponding geometric homology, and that cycles on Y surject onto $A_{n+1-k}(Y)$. If specialization of cycle classes is compatible with these maps, every geometrically homologically trivial $z \in \mathrm{CH}^k(\mathcal{X}_K)_{\mathbf{Q}}$ has an extension whose restriction to each component has zero geometric cycle class.

Proof. Choose an extension $\Theta \in \mathrm{CH}^k(\mathcal{X})_{\mathbf{Q}}$, and let $a \in A^k(Y)$ be its component restriction. Compatibility with specialization and homological triviality of z imply that $a \cap [Y]$ has zero geometric homology class. Injectivity makes it zero in $A_{n-k}(Y)$. Exactness gives $b \in A_{n+1-k}(Y)$ with $i^*i_*b = a$. Lift b to a rational cycle class β on Y and set

$$\Theta_0 = \Theta - i_*\beta.$$

The vertical correction leaves the generic class unchanged and kills its component restriction. This gives the required extension. □

For the semistable abelian models used in [4, Section 3.1], these algebraic inputs are supplied by Künnemann's results. To deduce zero actual curvature from this correction, one additionally needs the numerical-model criterion: a pure operational model character with zero pairing against all vertical testing cycles must have a global actual primitive, after the stated finite descent. The model argument and the analytic criterion are distinct steps. In higher-dimensional general models, establishing the appropriate exactness and the actual comparison is one of the substantive problems of this route.

4 The global pairing on the obstruction kernels

We return to X/F and the groups (1.2). Use the ordinary absolute values on F_v, with complex absolute value unsquared, and put

$$w_v = \frac{[F_v : \mathbf{Q}_v]}{[F : \mathbf{Q}]}.$$

At infinite places the local degree is 1 or 2. The normalization is fixed by

$$\sum_v w_v \log |a|_v = 0 \qquad (a \in F^\times). \tag{4.1}$$

The local correction supplies forms for representatives of classes in $\mathcal{H}^r$ and $\mathcal{H}^s$. To assemble their integrals, the construction also needs reciprocity, the principal-divisor law and good-place vanishing. The following formulation makes those additional requirements explicit.

Hypothesis 4.1 (Local height identities)**.** For disjoint rational cycles Z, W with classes in $\mathcal{H}^r(X)$ and $\mathcal{H}^s(X)$, respectively, the local numbers

$$\lambda_v(Z, W) = \int_{W_{F_v}^{\mathrm{an}}} \gamma_{Z,v}$$

at finite places, with matching complex Green integrals at infinite places, satisfy the following properties.

1. They are independent of the Green choices, symmetric in complementary degrees, and rational-bilinear on cycles in common disjoint position.
2. For each fixed disjoint pair, all but finitely many local numbers are zero.
3. If Z_0 and Z_1 are rationally equivalent and both avoid W, a moving chain over F gives $a \in F^\times$ and $N \in \mathbf{Z}_{>0}$ such that

$$\lambda_v(Z_1, W) - \lambda_v(Z_0, W) = -N^{-1} \log |a|_v \tag{4.2}$$

   at every place. The same element a and denominator N occur at every place. The corresponding assertion holds in the other variable.
4. Each class can be represented by a cycle avoiding any finite list of complementary cycles, and the preceding moving chains can be chosen over F.

In [3, Section 3], the supported-complex construction gives a point-residue formula on normal testing varieties. Its refined coefficients identify a in (4.2) as a product of algebraic field norms. Disjoint-polar Stokes gives part (1), and smooth good-reduction models with disjoint cycle closures give part (2). The paper states the actual-calculus, finite-trace and supported-complex premises for these arguments. The finite set in part (2) may depend on the pair of representatives.

Theorem 4.2 (A height on the common local kernels)**.** *Under Hypothesis 4.1, the formula*

$$h_X(z, w) = \sum_v w_v \lambda_v(Z, W) \tag{4.3}$$

defines a rational-bilinear pairing

$$\mathcal{H}^r(X) \times \mathcal{H}^s(X) \longrightarrow \mathbf{R},$$

independent of the disjoint representatives and of the Green forms, and symmetric when the complementary factors are interchanged. If the domains and local pairings are compatible with finite extension and $\lambda_u(Z_{E_u}, W_{E_u}) = \lambda_v(Z_{F_v}, W_{F_v})$ for $u \mid v$, the pairing is invariant under finite extension E/F on pulled-back classes.

4.1 Proof of the global construction

Proof. The sum is finite by part (2), and part (1) removes the Green choices. For fixed W, the change of Z in (4.2) changes the sum by

$$-N^{-1} \sum_v w_v \log |a|_v = 0$$

by (4.1). Thus the value is independent of the representative of the first class among cycles avoiding W. The same argument applies to the second class.

Let (Z_0, W_0) and (Z_1, W_1) be any two disjoint representative pairs of the same classes. Choose W' rationally equivalent to W_0 and W_1 which avoids both Z_0 and Z_1. The successive comparisons

$$(Z_0, W_0), \quad (Z_0, W'), \quad (Z_1, W'), \quad (Z_1, W_1)$$

preserve the height. This proves simultaneous independence. To prove additivity, move the finite collection of cycles in an asserted additive relation into common disjoint position. Part (1) then proves the relation for the local numbers and hence for their finite sums. Rational homogeneity is identical. Symmetry follows from local symmetry.

For the extension assertion, let $w_u = [E_u : \mathbf{Q}_v]/[E : \mathbf{Q}]$. Group the sum over places u above v. The local equality and the identity

$$\sum_{u|v} w_u = \frac{[F_v : \mathbf{Q}_v]}{[E : \mathbf{Q}]} \sum_{u|v} [E_u : F_v] = \frac{[F_v : \mathbf{Q}_v]}{[F : \mathbf{Q}]} = w_v$$

give equality of the two global sums, including the infinite places. □

Corollary 4.3 (The full complementary Chow groups)**.** *Assume Hypothesis 4.1 and a solution to Problem 1.1. Then* (4.3) *has the full domain* (1.1) *and retains every property proved in Theorem 4.2.*

Proof. The vanishing assertion in degree r gives $\mathcal{H}^r(X) = V^r(X)$, and that in degree s gives $\mathcal{H}^s(X) = V^s(X)$. The pairing and its proof are unchanged. □

If only the degree-r vanishing is known, the conclusion is instead the pairing $V^r(X) \times \mathcal{H}^s(X)$. This distinction is relevant, for example, when a one-cycle vanishing theorem supplies the full codimension-$n-1$ factor while codimension 2 remains restricted. In dimension 3 the two degrees coincide. The geometric domains, including their model and field hypotheses, are organized in [4].

Correspondence adjunction and equality with classical height constructions require the projection and model-comparison statements in [3, Sections 8–9]. They can be attached to the pairing on their stated domains. Positivity and arithmetic finiteness are subsequent questions about that pairing, not premises of Theorem 4.2.

5 Finite periods and the remaining programme

5.1 Comparison with a finite system of periods

Definition 5.1 (A finite detection criterion)**.** For fixed (X, K, k) and a specified rational subspace $V \subseteq \mathrm{CH}^k(X)_{\mathbf{Q}}$, a finite detection criterion consists of linear maps $P_1, \dots, P_m : V \to \mathbf{R}$, constructed independently of the individual class, and a proof that

$$\bigcap_{j=1}^{m} \ker P_j \subseteq \ker(\mathfrak{o}^k_{X,K}|_V). \tag{5.1}$$

It is faithful if equality holds.

Proposition 5.2 (Reduction to period vanishing)**.** *Under Hypothesis 2.2, a finite criterion as in Definition 5.1, together with $P_j|_V = 0$ for every j, gives strong Green forms for every prescribed cycle with class in V.*

Proof. Every class in V belongs to the left side of (5.1). Its original obstruction is therefore zero. Apply Proposition 2.3 to its prescribed representative. □

For a global application, suppose a finite set S of finite places is fixed such that all obstruction maps outside S vanish on $V^k(X)$. At each $v \in S$, suppose a faithful detector is defined on the localizations of global classes. Then

$$\mathcal{H}^k(X) = \ker\left(V^k(X) \longrightarrow \bigoplus_{v\in S} \mathbf{R}^{m_v}, \quad z \longmapsto (P_{v,j}(z_{F_v}))_{v,j}\right).$$

This equality follows by applying the faithful criterion at every place in S and using vanishing outside S. A sufficient criterion without the reverse inclusion gives only a subspace of $\mathcal{H}^k(X)$. In particular, a finite list of periods cannot replace the original kernel definition until its comparison has been established.

5.2 Construction, vanishing and arithmetic comparison

Problem 5.3 (The prescribed Green pair and its trace)**.** Prove Hypothesis 2.2 for every smooth projective X/K and every codimension. For the supported-complex method, construct G_Z on all of U_Z, retain the localized cycle coefficients in (2.1), and prove that changing the model or resolution changes Ω_Z by D of a global actual form. Construct the finite trace on the forms used by this proof with presentation independence, both differential compatibilities and all-test adjunction.

The decisive issue is the passage from expressions on covers to an actual form downstairs. A proper generically finite map can be used for algebraic intersection calculations, but the asserted form-level trace must be defined on the finite locus on which it is used. Likewise, an alteration need not become finite over the prescribed complement. A construction relying on that property must provide it for its particular map.

Problem 5.4 (The remaining homologically trivial classes)**.** Assume the local construction. Prove that the induced obstruction on (3.1) is zero, or, for the global goal, prove only (1.3). In a period approach, first establish (5.1) for the original curvature class and then compute its periods on the same Chow source.

A comparison with ℓ-adic, de Rham or Abel–Jacobi data must identify the obstruction-valued map, not just two abstract cohomology spaces. Homological triviality kills the cycle class; it does not by itself kill every secondary invariant. The missing inference is the vanishing of the particular secondary periods that detect $\mathfrak{o}$.

Problem 5.5 (Local residues and normalized global comparison)**.** Establish Hypothesis 4.1 from the proposed actual-form construction in every relevant codimension. In the principal transformation formula, retain the refined point coefficients and prove that the local scalar is the absolute value of one algebraic element of $F^\times$ at every place. Then prove correspondence adjunction and comparison with classical Beilinson–Bloch heights on the common domains, with the absolute normalization (4.1).

These are three different mathematical obligations. The first supplies the local obstruction and a Green pair. The second enlarges the subgroup on which curvature can be removed. The third makes the local numbers descend and identifies the resulting height. Once the local height identities are established, vanishing on the global source in the two complementary degrees is exactly what completes this obstruction route to the full pairing.

References

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